O K L A H O M A   S T A T E   U N I V E R S I T Y

S C H O O L   O F   E L E C T R I C A L   A N D   C O M P U T E R   E N G I N E E R I N G
S C H O O L O F M E C H A N I C A L A N D A E R O S P A C E E N G I N E E R I N G

ECEN 4413/MAE 4053
Automatic Control Systems
Spring 2008

Midterm Exam #2

For all students, choose any four out of five problems.
Please specify which four listed below to be graded
:
1)_____; 2)_____; 3)_____; 4)_____;

Name : ______________________________

E-Mail Address:_______________________________________

Problem 1:
The equations that describe the dynamics of a motor control system are

$$e_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + K_b \frac{d\theta_m(t)}{dt}$$

$$T_m(t) = K_i i_a(t)$$

$$T_m(t) = J \frac{d^2\theta_m(t)}{dt^2} + B\frac{d\theta_m(t)}{dt} + K\theta_m(t)$$

$$e_a(t) = K_a e(t)$$

$$e(t) = K_s\left[\theta_r(t) - \theta_m(t)\right]$$

a) Assign the state variables as $x_1(t) = \theta_m(t)$, $x_2(t) = d\theta_m(t)/dt$, and $x_3(t) = i_a(t)$. Express the state space representation in the form of

$$\frac{dx(t)}{dt} = Ax(t) + B\theta_r(t), \quad \theta_m(t) = Cx(t).$$

b) Develop a corresponding state diagram.

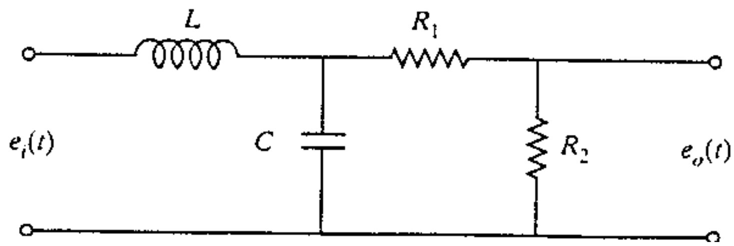
c) Find the closed-loop transfer function, $H(s) = \Theta_m(s)/\Theta_r(s)$.

Problem 2:

For the circuit diagram shown below, derive its state space representation in the form of

$$\dot{x}(t) = Ax(t) + bu(t)$$
$$y(t) = cx(t) + du(t)$$,

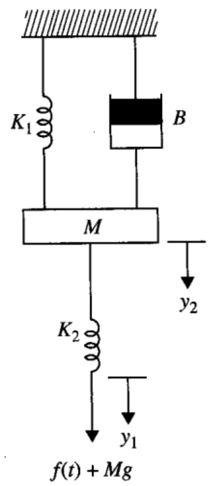
where input $u(t) = e_i(t)$ and output $y(t) = e_o(t)$ is the voltage across the resistor R_2. Develop a corresponding state diagram.



Problem 3:
For the mechanical system shown below, derive its state space representation in the form of

$$\dot{x}(t) = Ax(t) + bu(t)$$
$$y(t) = cx(t) + du(t)$$

where input is a force $f(t)$ pulling mass M downward (i.e., $u(t) = f(t)$) and output $y(t) = y_2(t)$. Please ignore the effect of gravitational force, Mg. Develop a corresponding state diagram.



Problem 4:

Given

$$\dot{x} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

with $x(0) = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$ and $u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$. Find the solution, $x(t)$.

Problem 5:

Find the solution of $\dot{x}(t) = Ax(t) + Bu(t)$, where

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

with

$$x(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{ and } u(t) = 1 \text{ for all } t \geq 0 \text{ (unit step function).}$$